2014

PHYSICS

(Major)

Paper : 5.3

(Quantum Mechanics and Astrophysics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Write the answers to the two Groups in separate books

GROUP—A

(Quantum Mechanics)

(*Marks* : 40)

1. Choose the correct answer from the given alternatives (any *four*) : 1×4=4

(a) The failure of the classical wave theory to account the distribution of energy in the spectrum of a blackbody radiation was due to the assumption that, radiation energy is

(i) continuous

(ii) discrete

(iii) mixture of continuous and discrete

(iv) electromagnetic

(b) Davisson and Germer experiment suggests that the electron is

(i) a particle

(ii) a wave

(iii) partly wave and partly particle

(iv) None of the above

(c) Matter wave functions are

(i) periodic functions

(ii) real functions of x and t

(iii) imaginary functions of x and t

(iv) complex functions of x and t

(d) The value of

$$\left[\hat{x}, \frac{\partial}{\partial x}\right]$$

is

(i) 1

(ii) -1

(iii) $i\hbar$

(iv) $-i$

(e) The wave function $\psi(x)$ is well-behaved if $\psi(x)$ is single-valued and

(i) $|\psi(x)| \to 0$ as $x \to \pm\infty$

(ii) $|\psi(x)| \to 0$ as $x \to 0$

(iii) $|\psi(x)| \to \infty$ as $x \to \pm\infty$

(iv) $|\psi(x)| \to$ finite value as $x \to \pm\infty$

2. Answer any *three* questions : 2×3=6

(a) Explain why we do not observe quantum effects in case of a fast moving cricket ball.

(b) What is the physical significance of the wave function $\psi(x, t)$?

(c) Write down the time-dependent Schrödinger equation for a particle of mass m moving in a potential $v(\vec{r}, t)$.

(d) If ψ is an eigenfunction of both $\hat{\alpha}$ and $\hat{\beta}$, then prove that $[\hat{\alpha}\hat{\beta} - \hat{\beta}\hat{\alpha}] = 0$.

(e) Distinguish between a classical and a quantum harmonic oscillator.

3. Answer any *two* questions : 5×2=10

(a) An incident X-ray photon of frequency ν_0 is scattered by a free electron at rest through an angle ϕ. Using relativistic expression of electron energy, show that the change in the wavelength of the photon is given by

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

where m_0 = rest mass of the electron, h is Planck's constant and c is velocity of light in vacuum. 5

(b) State and explain the de Broglie's hypothesis.

Calculate the de Broglie wavelength of electrons of energy 10^4 eV and compare it with the wavelength of electromagnetic radiation for which the photon has the same energy. 2+3=5

(c) Using uncertainty relation, show that an electron cannot reside inside a nucleus. A nucleon is confined to a nucleus of diameter 5×10^{-14} m. Calculate the minimum uncertainty in the momentum of the electron and the minimum kinetic energy of the electron. Given, $m_e = 9\cdot1\times10^{-31}$ kg. 2+2+1=5

(d) What do you mean by expectation value of an operator in quantum mechanics? If $\psi(x) = Ae^{-m\omega x^2/\hbar}$, find the expectation values of momentum. 2+3=5

4. Answer *either (a)* and *(b) or (c)* and *(d)* : 5×2=10

(a) A particle on the x-axis has the wave function $\psi(x) = cx^2$ between $x = 0$ and $x = 2$. Normalize the wave function over the interval. Find the probability that the particle can be found between $x = 0\cdot5$ and $x = 0\cdot6$. 2+3=5

(b) State and prove Ehrenfest's theorem. 5

Or

(c) Describe briefly the experiment of G. P. Thomson on the diffraction of electrons.

In the experiment, electrons accelerated by a potential difference of 20 kilo volts are diffracted by a thin metal foil. Calculate the Bragg angle for its first-order diffraction from a set of crystal planes which are 2·0 Å apart. 3+2=5

(d) Show that if $\psi_1(\vec{r})$ and $\psi_2(\vec{r})$ are two independent solutions of the Schrödinger's equation, then

$$\psi(\vec{r}) = a_1\psi_1(\vec{r}) + a_2\psi_2(\vec{r})$$

is also a solution of the Schrödinger's equation. What does it imply? 4+1=5

5. Answer *either (a)* and *(b) or (c)* and *(d)* : 5×2=10

(a) Write down the Schrödinger equation for a linear harmonic oscillator. What are the eigenvalues and the eigenfunctions of the Hamiltonian of a linear harmonic oscillator? Explain the significance of zero-point energy of the oscillator. 1+2+2=5

(b) State and explain the complementary principle of Niels Bohr. What conclusion can be drawn from the result of γ-ray microscope experiment? 3+2=5

Or

(c) Derive the continuity equation from the time-dependent Schrödinger equation of a particle moving in a real potential. Give the physical interpretation of the continuity equation you derive. 4+1=5

(d) What are conjugate variables in quantum mechanics? Give an example of any one pair of conjugate variables and obtain their commutation relation. 1+1+3=5

GROUP—B

(Astrophysics)

(*Marks* : 20)

6. Answer any *three* of the following : 2×3=6

(a) Define the right ascension and declination of a celestial object.

(b) What do you mean by sidereal time? How is it different from the solar time?

(c) The apparent and absolute magnitudes of a star are +8·6 and +11·4 respectively. Find its distance in parsec.

(d) Find the meridian Zenith distance of Vega ($\partial = +38°44'$) at New Delhi ($\phi = 28°22'N$). Neglect the effect of atmospheric refraction.

7. Answer any *two* of the following : 4×2=8

(a) What is Hertzsprung-Russell diagram? Draw a neat sketch of the H-R diagram showing the position of the main sequence stars with the Sun and the white dwarfs.

(b) Write down the various equations of PP–I, PP–II and PP–III chain reactions that convert four H nuclei into a He nucleus.

(c) Derive a relation between the apparent and absolute magnitudes of a star.

8. Write a short note on any *one* of the following : 6

(a) Pulsars

(b) Black holes

(c) Evolution of the Universe

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[The following data can be used when required :

$[e = 1·6 \times 10^{-19}$ C, $c = 3 \times 10^{8}$ m/s

$h = 6·6 \times 10^{-34}$ J s, $m_e = 9·1 \times 10^{-31}$ kg

$m_p = 1·67 \times 10^{-27}$ kg]