2014

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 30)

1. Answer the following questions : 1×4=4

(a) What is the argument of $-3i$?

(b) Express the number $-\sqrt{6}-\sqrt{2}i$ in polar form.

(c) Plot the number $e^{\left(1+\frac{\pi}{4}i\right)}$.

(d) Find the real part of $\frac{1+z}{1-z}$.

2. (a) Find and plot all the roots of $(1-i)^{\frac{1}{4}}$. 2

(b) Prove that, $\arg\left(\frac{z_1}{z_2}\right)=\arg z_1-\arg z_2$. 2

3\. (a) Check the analyticity and hence find the derivative of the function $f(z) = \sin z$. 4

(b) Using Cauchy's integral formula, evaluate

$$\oint_C \frac{z^2}{(z-1)^3} dz$$

where C is a circle given by $|z| = 2$. 4

Or

Find Taylor series expansion about the origin for $f(z) = \ln(1+z)$.

4\. (a) Define pole, simple pole, isolated singularity and essential singularity. 2

(b) Find Laurent expansion for the function

$$f(z) = \frac{\sin z}{z^4}$$

about $z_0 = 0$ and hence classify the singularity and calculate the residue. 5

Or

Derive Cauchy's integral formulas. 7

5\. Calculate the residues of $f(z) = \frac{z^2}{(1+z^2)^2}$ and evaluate $\int_0^\infty \frac{x^2 dx}{(1+x^2)^2}$. 3+4=7

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

6\. Answer the following questions : 1×4=4

(a) What is the nature of orbit for an object moving under the influence of an inverse square law force with total energy $E < 0$?

(b) A system of 5 particles has 12 equations of constraints and requires 3 generalized coordinates. Are the constraints holonomic or non-holonomic?

(c) Write down the Lagrange's equation of motion for a non-conservative system.

(d) What is the expression of Hamiltonian of a system in spherical polar coordinates?

7\. (a) Show that angular momentum is a constant of central force motion. 2

(b) What are generalized forces and generalized momenta? 2

Or

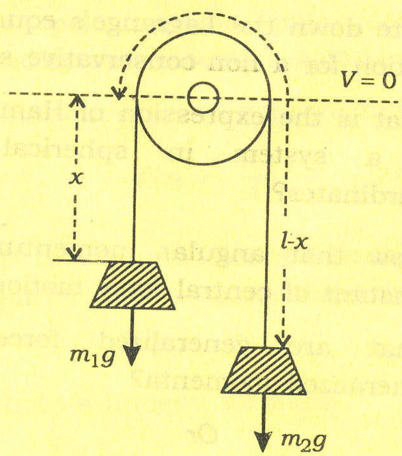
Determine the Hamiltonian of a system if its Lagrangian is given by $L = \frac{1}{5}\dot{q}^2 + \alpha(q - q_0)^2$, where α is a constant. 4

8. Answer any *three* of the following questions : 4×3=12

(a) Obtain the general differential equation of a central orbit.

(b) Using the variational principle, show that the shortest distance between two points in a plane is a straight line.

(c) Referring to the figure given below, consider a system of two masses m_1 and m_2 tied together with a light inextensible cord of length l passing round a frictionless pulley :



Find the equation of motion for this system employing Lagrange's equation.

(d) A simple pendulum hangs from the ceiling of an elevator which is moving down with a constant acceleration a. Obtain the Hamiltonian and hence the equation of motion of the simple pendulum.

(e) Deduce the Hamilton's canonical equations in terms of Poisson's brackets and show that Poisson's bracket of two constants of motion is itself a constant of motion.

9. Answer any *two* questions : 5×2=10

(a) A particle follows a spiral orbit given by $r = ae^{b\theta}$ under the influence of a central force, where a and b are constants. Obtain the force law.

(b) Show that the total energy of a particle of mass m acted upon by a central force is given by

$$E = \frac{L^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r)$$

where $V(r)$ is the potential energy, L the angular momentum and (r, θ) the polar coordinates of the particle; $u = 1/r$.

(c) Starting from the Hamilton's principle, deduce the Lagrange's equations of motion.

(d) Using Hamilton's canonical equations, derive the equation of motion of a particle moving in a force field in which the potential is given by $V = -k / r$, where k is positive.

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