3 (Sem 4) PHY M1

2015

PHYSICS

(Major)

Theory Paper : M-4.1

Full Marks – 60

Time – 2½ hours

The figures in the margin indicate full marks
for the questions.

GROUP–A

1. Answer any *four* of the following questions :

$1 \times 4 = 4$

(a) Define ordinary point of a second order differential equation.

(b) What is the value of $P_n(t)$?

(c) Define total probability.

(d) Under what condition does the Gaussian distribution become normal distribution ?

[Turn over

(e) Give an example where Legendre polynomial is used in Physics.

(f) What is meant by mean deviation ?

2. Answer any *three* of the following questions :

$$2 \times 3 = 6$$

(a) Check whether Frobenius method can be applied to the following equation or not.

$$\frac{d^2 y}{dx^2} - \frac{9y}{x^3} = 0$$

(b) Prove that $P_n^m(-x) = (-1)^{n+m} P_n^m(x)$.

(c) What is the probability that the ace of spaces will be drawn from a deck of cards at least once in 104 consecutive trials ?

(d) Prove the following recurrence relation :
$$2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x).$$

(e) Find the degree and order of the following equation :

$$\left(\frac{d^2 y}{dx^2}\right)^{2/3} = \left(y + \frac{dy}{dx}\right)^{1/2}$$

3. Answer any *two* of the following questions :

$$5 \times 2 = 10$$

(a) Establish the following recurrence formula for Legendre polynomial $P_n(x)$

$$nP_n(x) = (2n-1) xP_{n-1}(x) - (n-1) P_{n-2}(x). \quad 5$$

(b) Find the singularity of the differential equation $(1-x^2)y'' + xy' + y = 0$ and discuss the nature of the singularity. 5

(c) Find the probability of almost 5 defective fuses to be found in a box of 200 fuses, if experience shows that 2% of such fuses are defective.

5

(d) Show that $\displaystyle\int_{-1}^{+1} x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1}.$ 5

4. Answer any *two* of the following : $10 \times 2 = 20$

(a) (i) Using the following definition of the Legendre polynomials $P_n(x)$,

$$\left(1 - 2xt + t^2\right)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x)$$

show that $|P_n \cos(\theta)| \leq |$ 6

(ii) Prove the recurrence relation
$$xP_n' - P_{n-1}' = nP_n' \qquad 4$$

(b) (i) Find the indicial equation of the Hermite equation $\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2xy = 0$.   5

(ii) Show that the generating function for Hermite polynomial $H_n(x)$, for integral n, and real values of n is given by

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x). \qquad 5$$

(c) (i) What is Gaussian distribution ?   2

(ii) Define standard deviation.   2

(iii) Prove the theory of compound probability.   5

(iv) Write one property of normal distribution.   1

(d) (i) Show that $\displaystyle\int_{-1}^{+1} P_n(x)\, P_m(x)\, dx = 0$

where $P_n(x)$ and $P_m(x)$ are solutions of the Legendre differential equation.   6

(ii) Show that

$$H_0(x) = 1 \text{ and } H_1(x) = 2x. \qquad 2+2=4$$

GROUP – B

5. Answer any *two* of the following :   1×2=2

(a) What is the function of control unit ?

(b) Give the logical AND operation.

(c) What is a string ?

6. Answer any *two* of the following :   2×2=4

(a) Define with example an operating system.

(b) What are control statements ? Give examples.

(c) Write a syntax in C/FORTRAN. How is an array declared in a program ?

7. Answer any *one* of the following :   4

    (a) Write a program in FORTRAN / C / C⁺⁺ to find the sum of first n natural numbers.

    (b) Draw a flow chart to find the factorial of a number.

8. Answer any *one* of the following questions :

$$10 \times 1 = 10$$

    (a) Describe with a block diagram for the different functional units of a digital computer.   10

    (b) Write the algorithm and draw the flow chart to find the largest of N numbers. 5+5=10