

### 3 (Sem-2) PHY M 1

2014

PHYSICS

( Major )

Paper : 2.1

Full Marks : 60

Time : 2½ hours

*The figures in the margin indicate full marks  
for the questions*

GROUP—A

( **Mathematical Methods II** )

( Marks : 35 )

1. Answer the following questions : 1×4=4

(a) If  $\vec{R}(u) = \frac{d}{du} \vec{S}(u)$ , find  $\int_a^b \vec{R}(u) du$ .

(b) If the surface integral of  $\vec{A}$  over a closed surface  $S$  vanishes, evaluate  $\vec{\nabla} \cdot \vec{A}$ .

(c) Write the transformation equations between Cartesian coordinates and spherical coordinates.

(d) Give the graphical representation of the Dirac delta function  $\delta(x - x_0)$ .

2. Answer the following questions :  $2 \times 3 = 6$

(a) If  $\vec{E} = -\vec{\nabla}\phi$ , evaluate  $\oint_C \vec{E} \cdot d\vec{r}$ , where  $\phi$  is a scalar function of  $r$ .

(b) Show that

$$\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV = \int_S (\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi) d\vec{S}$$

(c) For an orthogonal curvilinear coordinate systems, show that

$$\hat{e}_2 = h_3 h_1 \vec{\nabla} u_3 \times \vec{\nabla} u_1$$

where symbols stand for usual meanings.

3. Using Green's theorem in plane, find

$$\oint_C [(xy + y^2) dx + x^2 dy]$$

where  $C$  is the closed curve of the region bounded by  $y = x$  and  $y = x^2$ .

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Or

A fluid of density  $\rho(x, y, z, t)$  moves with velocity  $\vec{v}(x, y, z, t)$ . If there is neither any source nor any sink, using divergence theorem, show that

$$\vec{\nabla} \cdot (\rho \vec{v}) + \frac{\partial \rho}{\partial t} = 0$$

4. Answer either (a) or [(b) and (c)] :

Either

(a) Prove that

$$\oint_C \vec{A} \cdot d\vec{\lambda} = \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}$$

where  $C$  is the curve bounding the surface  $S$ . Hence find  $\oint \vec{r} \cdot d\vec{r}$ .  $8+2=10$

Or

(b) Express  $\vec{\nabla}\phi$  in the orthogonal curvilinear coordinate system. 7

(c) Find  $\Gamma(-\frac{5}{2})$  provided  $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ . 3

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5. Answer either [(a) and (b)] or [(c) and (d)] :

Either

(a) Prove that

$$\int_V (\vec{\nabla} \times \vec{B}) dV = \int_S (\hat{n} \times \vec{B}) dS$$

Where  $V$  is the volume enclosed by the surface  $S$  and  $\hat{n}$  is the unit normal vector to the plane of  $dS$ . 6

(b) Show that

$$\int_{-\infty}^{+\infty} \delta(a-x) \delta(b-x) dx = \delta(a-b) \quad 4$$

Or

(c) Define Gamma function and establish

$$\Gamma(n+1) = n \Gamma(n) \quad 1+4=5$$

(d) Find the square of the elemental length in cylindrical coordinates and determine the corresponding scale factors. 4+1=5

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GROUP—B

( Properties of Matter )

( Marks : 25 )

6. Answer the following questions : 1×3=3

(a) Draw the stress-strain graph indicating the proportional limit and the yield point.

(b) State the principle on which the action of the split tip of a fountain pen's nib is based.

(c) A body is being moved horizontally through a viscous medium. What is the angle between the viscous force on the body and its weight?

7. Find the maximum length of a wire that can be suspended without breaking. Given that its breaking stress and density are  $7.2 \times 10^8 \text{ N/m}^2$  and  $7.2 \times 10^3 \text{ kg/m}^3$ . Take  $g = 10 \text{ m/s}^2$ . 2

8. Answer any two of the following questions : 5×2=10

(a) A body is subjected to a stress. Show that the potential energy stored in the unit volume of the body is

$$\frac{1}{2} \times \text{stress} \times \text{strain}$$

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(b) Find an expression for an excess pressure at a point on a curved liquid surface. Hence find the excess pressure for a spherical soap bubble. 4+1=5

(c) A particle of mass  $m$  is moving through a viscous medium. If the viscous force varies linearly with instantaneous velocity  $v$ , find the expression for  $v$  as a function of time  $t$ . The initial velocity is  $v_0$ . 5

9. Answer either [(a) and (b)] or [(c) and (d)] of the following questions :

*Either*

(a) Derive an expression for the twisting couple per unit twist of a rod of the length  $l$ , the radius  $r$  and the rigidity modulus  $\eta$  fixed at one end. 7

(b) Calculate the terminal velocity of an air bubble of radius  $10^{-5}$  m rising in water of viscosity  $10^{-3}$  Ns/m<sup>2</sup>. Density of water is  $10^3$  kg/m<sup>3</sup> and that of air is negligible. 3

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Or

(c) Derive the relation

$$Y = 3K(1 - 2\sigma)$$

where  $Y$ ,  $K$  and  $\sigma$  are Young's modulus, bulk modulus and Poisson's ratio respectively. 7

(d) Calculate the work done against surface tension in blowing a soap bubble from a radius of 10 cm to 20 cm, if the surface tension is  $25 \times 10^{-3}$  N/m. 3

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