Total No. of printed pages = 6

3 (Sem 6) MTH M5

2015

MATHEMATICS

(Major)

Theory Paper : M-6.5

(Graph and Combinatorics)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times7=7$

(a) How many functions are there from a set with 3 elements to a set with 5 elements ?

(b) How many ways are there to draw a club or a spade from a pack of cards ?

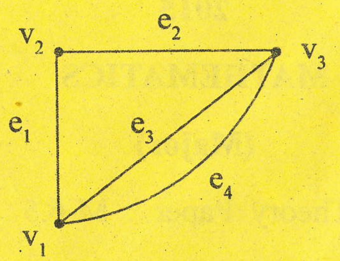
(c) Draw a simple graph having four vertices each of degree two.

[Turn over

(d) Define the union of two graphs.

(e) Determine if the walk

$(v_2, e_2, v_3, e_3, v_1, e_4, v_3)$ is a path



(f) What is meant by the length of a walk ?

(g) Is the following statement true ?

"In any graph, the number of odd vertices is even."

2. Answer the following questions : $2\times4=8$

(a) State the rule of sum in the theory of counting.

(b) Find the number of subsets of the set

$\{1, 2, 3, 4, \ldots\ldots, n\}$.

(c) Define a complete bipartite graph. Draw a complete bipartite graph on 2 and 4 vertices.

(d) Does there exist a simple graph with five vertices having degrees 2, 2, 4, 4, 4 ? Justify.

3. Answer any *three* parts of the following : $3\times5=15$

(a) Give combinational proofs of the following identities : $2+3=5$

(i) $C(n, r) = C(n, n-r)$

(ii) $C(n+1, r) = C(n, r) + C(n, r-1)$

(b) (i) How many selections can be made from 3 white balls, 4 green balls, 1 red ball, 1 black ball ; if at least one must be chosen ?

(ii) In how many ways can a person invite one or more of his 5 friends to a party ? $3+2=5$

(c) (i) Draw the graphs K_4 and $K_{2,3}$.

(ii) How many vertices are there in a graph with 15 edges if each vertex is of degree 3 ? $2+3=5$

(d) Define a path.

If a graph G contains exactly two vertices of odd degree, show that there exists a path between these two vertices. 1+4=5

(e) Define a tree.

If in a graph G, there is a unique path between every pair of vertices, show that G is a tree. 1+4=5

4. (a) How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 16$, where each $x_i \geq 2$? 3

(b) How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$, where $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0$? 4

(c) What is the probability that exactly one cell is empty if ten identical balls are distributed randomly into five distinct cells ? 3

5. (a) Prove that a connected graph G with n vertices is a tree if and only if G contains (n − 1) edges. 5

Or

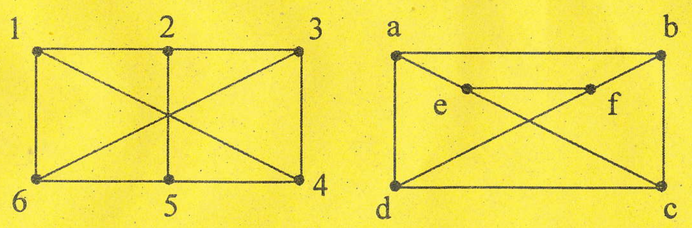
Prove that a connected graph G remains connected after removing an edge e from G if and only if e is in some cycle in G. 5

(b) Define

(i) a bridge in a graph,

(ii) a separable graph. 1+1=2

(c) Define graph isomorphism. Examine if the following two graphs display an isomorphism between them. 1+2=3



6. (a) (i) Define an Eulerian graph and a Hamiltonian graph.

(ii) Give an example of a graph which is Hamiltonian, but not Eulerian.

(iii) Give an example of a graph which is Eulerian but not Hamiltonian.

1+1+1+1=4

(b) If a connected graph G is Eulerian, prove that every vertex of G has even degree. 6

Or

Prove that there is always a Hamiltonian path in a directed complete graph. 6