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3(Sem 6) MTH M4

2015

MATHEMATICS

(Major)

Theory Paper : M-6.4

(Discrete Mathematics)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed. $1\times7=7$

(a) If n is a positive integer such that n^3+1 is a prime, then find the value of n.

(b) For all integers $n \geq 0$, $7^n - 1$ is divisible by 6. (State whether true or false).

(c) State Euclid's theorem on prime numbers.

(d) Find all integers $k \geq 2$ such that $7 \equiv k \pmod{k^2}$.

(e) If n is a positive integer such that gcd (n, 9) = 1, then $n^{18} - 1$ is not divisible by 9. (State whether true or false).

(f) State Fermat's Little Theorem.

(g) State the condition for which the linear Diophantine equation $ax + by = c$ has an integral solution.

2. Answer the following questions : 2×4=8

(a) If a and b are positive integers such that gcd (a, b) = 1, then show that gcd $(a+b, a-b) = 1$ or 2.

(b) Find the remainder when $\underline{|17}$ is divided by 19.

(c) Show that if $x^2 + y^2 = z^2$, then one of x, y is $\pm 1 \pmod 4$ and the other is $0 \pmod 4$.

(d) Find all integral solution of the following linear Diophantine equation $8x - 10y = 42$.

3. Answer the following questions : 5×3=15

(a) For any integer $n \geq 2$, if p divides $a_1, a_2 \ldots a_n$, then prove that p divides one of the integers $a_1, a_2, \ldots, a_n$, where p is a prime number. Applying this result, show that 12 is not a prime number.

Or

If p_n is the nth prime, then prove that

$$\frac{1}{p_1} + \frac{1}{p_2} + \ldots\ldots + \frac{1}{p_n}$$ is not an integer.

(b) Solve the linear congruence

$$6x \equiv 15 \pmod{21}$$

Or

Determine the integer in the unit place of $17^{17^{17}}$.

(c) If $p^c|n$, $p^{c+1} \nmid n$ where p is a prime of the form $4k + 3$ and c is odd, then prove that n has no representation as the sum of two squares.

4. (a) Answer either (i) or (ii) : 10

(i) (1) If the integer $n > 1$ has the prime factorization

$n = p_1^{k_1} p_2^{k_2} \ldots\ldots p_r^{k_r}$ then show that

$$\tau(n) = \prod_{i=1}^{r} (k_i + 1).$$

Hence show that τ is a multiplicative function. 5

(2) Define Mobius μ function. Show that μ is a multiplicative function. If n is a positive integer such that $n \geq 3$, show that

$$\sum_{k=1}^{n} \mu(\lfloor k) = 1.$$ 3+2=5

(ii) Define Euler's phi-function. Find $\phi(20)$. If p is a prime and n is a positive integer, then prove that

$$\phi(p^n) = p^n\left(1 - \frac{1}{p}\right).$$ 1+2+7=10

(b) Answer either (i) or (ii) : 10

(i) (1) Let p be "6 is a real number", q be "2+4 = 9" and r be "sum of two even integers is even". Then find the truth value of the following statement forms : 5

(i) $p \rightarrow (q \wedge r)$

(ii) $(p \wedge q) \vee (p \wedge r)$

(iii) $(p \rightarrow (\sim q \vee r)) \wedge \sim (q \vee (p \leftrightarrow r))$

(2) State the principle of substitution. Using the principle, show that the following statements formula is a tautology :

$$(p_1 \wedge \sim p_2) \rightarrow ((\sim p_3 \wedge p_4) \rightarrow ((p_1 \wedge \sim p_2) \wedge (\sim p_3 \wedge p_4)))$$ 5

(ii) (1) Using truth table, verify the following :

$$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$$ 5

(2) Write the truth tables for the connectives 'NOR' and 'NAND'. Show that each of the connectives alone forms an adequate system. 5

(c) Answer either (i) or (ii) : 10

(i) (1) Express the following Boolean expression in disjunctive normal form (DNF) and conjunctive normal form (CNF) :

$(x + y + z)\,(xy + x'z)'$ 5

(2) Find a switching circuit which realizes the switching function f given by the following switching table : 5

x	y	z	f (x, y, z)
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

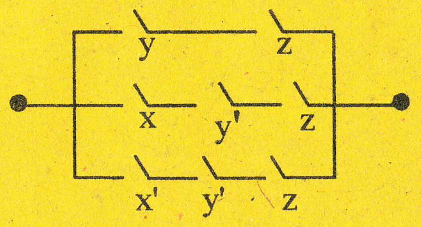
(ii) (1) Simplify the Boolean expression :

$(x + y)\,(x + z)\,(x'\,y')'$

Express the following Boolean expression in conjunctive normal form (CNF) in the variables present in the expression :

$x' + yz.$ 2½+2½=5

(2) Consider the following switching circuit :



Find a Boolean expression which represents the circuit. Also, draw a simpler equivalent circuit for the above circuit. 3+2=5