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3 (Sem 6) MTH M2

2015

MATHEMATICS

(Major)

Theory Paper : M-6.2

(Numerical Analysis)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

(a) Select the correct answer :

If x is the true value of a quantity and x_1 is its approximate value, then the relative error is

(i) $\left|\frac{x_1 - x}{x_1}\right|$

(ii) $\left|\frac{x - x_1}{x}\right|$

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(iii) $\left|\frac{x_1}{x}\right|$

(iv) $\left|\frac{x}{x_1 - x}\right|$

(b) Fill up the blank :

Approximate value of $\frac{1}{3}$ are given as 0.3, 0.33 and 0.34. Out of these the best approximation is ——.

(c) Define the term 'Absolute error'.

(d) What is the Kth difference of a polynomial of degree K ?

(e) Write the relationship between the operator E and the differential operator D.

(f) Choose the correct answer :

$\Delta \nabla =$

(i) $\nabla \Delta$

(ii) $\nabla + \Delta$

(iii) $\nabla - \Delta$

(iv) None.

(g) What is the degree of the approximating polynomial corresponding to trapezoidal rule and Simson's $\frac{1}{3}$rd rule ?

2. Answer the following questions : 2×4=8

(a) Using normalized floating point representation of real numbers,

Subtract ·9432 E–4 from ·5452 E–3

(b) With the usual notations, show that

$(1 + \Delta)(1 - \nabla) = 1$

(c) How do you choose the 'proper' interpolation formula for numerical differentiation ?

(d) f(x) is given by

x	:	0	0.5	1
f(x)	:	1	0.8	0.5

Using trapezoidal rule find the value of $\int_0^1 f(x)\,dx$.

3. Answer the following questions :

(a) Round off the number 37.46235 to four significant figures and compute Absolute error and Relative error. 1+4=5

(b) (i) Evaluate : $\Delta \tan^{-1} x$

(ii) Prove that $e^x = \left(\frac{\Delta^2}{E}\right) e^x . \frac{Ee^x}{\Delta^2 e^x}$;

the interval of differencing being h. 2+3=5

Or

A second degree polynomial passes through (0, 1), (1, 3), (2, 7) and (3, 13). Find the polynomial.

(c) Use Newton's forward interpolation formula

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots,$$ where

$$u = \frac{x - x_0}{h},$$ to establish the formula

$$\left(\frac{d^2y}{dx^2}\right)_{x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \frac{137}{180}\Delta^6 y_0 + \ldots\ldots\right]$$ 5

Or

Evaluate :

$\int_0^1 \frac{dx}{1+x^2}$ by using Simson's three-eighth formula. Hence, obtain the approximate value of π.

4. Answer either (a) or (b) :

(a) (i) What is meant by divided difference ? Prove that the divided differences are symmetrical in all their arguments. 1+4=5

(ii) Given :

$\log_{10}654 = 2{\cdot}8156$, $\log_{10}658 = 2{\cdot}8182$,

$\log_{10}659 = 2{\cdot}8189$, $\log_{10}661 = 2{\cdot}8202$,

find by using Lagrange's formula, the value of $\log_{10}656$. 5

(b) (i) Write briefly when the central difference formulae are used.

Derive Gauss's forward interpolation formula from Newton's forward interpolation formula. 2+4=6

(ii) Using Gauss's backward formula, estimate the population of a town for the year 1974 from the following data : 4

Year	: 1939	1949	1959	1969	1979	1989
Population : (in thousands)	12	15	20	27	39	52

5. Answer either (a) or (b) :

(a) (i) Find $f'(5)$ from the following table : 5

x :	0	2	3	4	7	9
f(x) :	4	26	58	112	466	922

(ii) A curve is drawn to pass through the points given by the following table :

x :	1	1·5	2	2·5	3	3·5	4
y :	2	2·4	2·7	2·8	3	2·6	2·1

Estimate the area bounded by the curve, x-axis and the line $x = 1$, $x = 4$. 5

(b) (i) What do you mean by numerical integration ? How to solve the problem of numerical integration ? 1+2=3

(ii) Show that the co-efficients of Newton-Cote's formula are symmetric from both the ends. 7

6. Answer either (a) or (b) :

(a) (i) Establish the Newton-Raphson formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Mention two situations where the formula fails to give a solution. 3+2=5

(ii) Find a root of the equation $x^3 - 2x - 5 = 0$, using Secant method correct to three decimal places. 5

(b) (i) Explain the bisection method with suitable diagram. Why bisection method is not applied to evaluate a double root of an equation. 4+1=5

(ii) Evaluate $\sqrt{12}$ to five decimal places by Newton-Raphson method. 5