2014

MATHEMATICS

(Major)

Paper : 6.4

(Discrete Mathematics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : $1\times7=7$

(a) State Peano's axioms.

(b) For any integer n, $4 / (n^2+2)$.

(State whether True or False)

(c) If a, b, c are positive integers, such that $\gcd(a, b, c)=1$. Then what will be the $\gcd(a, b)$ and $\gcd(a, c)$?

(d) State Chinese remainder theorem.

(e) Find all integers $k \geq 3$, such that $5 \equiv k^2 \pmod{k}$.

(f) Consider the congruence

$$4x \equiv 6 \pmod 4$$

Find out the correct statement.

The given congruence has

(i) unique solution

(ii) exactly two solutions

(iii) no solution

(iv) exactly four solutions

(g) The equation $18x + 12y = 2$ has no integral solution. Justify the statement.

2. Answer the following questions : 2×4=8

(a) If p is a prime and p / ab, then prove that either p / a or p / b.

(b) Find the remainder when 7^{30} is divided by 4.

(c) Find all solutions of the Diophantine equation $3x + 2y = 6$.

(d) Find all primitive solutions of $x^2 + y^2 = z^2$ in which $x = 40$.

3. Answer the following questions : 5×3=15

(a) If a and b are integers with $b > 0$, then show that there exists unique integers q and r satisfying

$$a = bq + r, \; 0 \le r < b$$

Or

If a and b are two non-zero integers, show that there exist integers x and y such that $\gcd(a, b) = ax + by$.

(b) Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod p$, where p is an odd prime, has a solution if and only if $p \equiv 1 \pmod 4$.

Or

If p is a prime and a is an integer not divisible by p, prove that

$$a^{p-1} \equiv 1 \pmod p$$

Hence show that for every integer a, $a^p \equiv a \pmod p$.

(c) If p is a prime of the form $4k + 1$, then prove that there exists a solution in integers x, y, m of $x^2 + y^2 = mp$, with $0 < m < p$.

4. (a) Answer either (i) or (ii) : 10

(i) (1) If p is a prime, prove that

$$\phi(p^k) = p^k - p^{k-1}$$

for any positive integer k. For $n > 2$, show that $\phi(n)$ is an even integer. 3+2=5

(2) State Möbius inversion formula. If the integer $n > 1$ has the prime factorization $n = p_1^{k_1} p_2^{k_2} \cdots p_s^{k_s}$, then prove the following :

$$\sum_{d/n} \mu(d)\sigma(d) = (-1)^s p_1 p_2 \cdots p_s$$

2+3=5

(ii) (1) Find the remainder when 35^{33} is divided by 24. 5

(2) Define the arithmetic function τ. Evaluate $\tau(180)$. If n is a square-free integer having r prime factors, prove that

$$\tau(n) = 2^r$$

1+2+2=5

(b) Answer either *(i)* or *(ii)* : 10

(i) (1) Examine if the following statement forms are tautologies : 5

$$(p \wedge q) \wedge (\sim (p \vee q))$$

$$(p \leftrightarrow q) \leftrightarrow ((p \to q) \wedge (q \to p))$$

(2) What do you mean by an adequate system of connectives? Show that $(\sim, \wedge)$ is an adequate system of connectives. 2+3=5

(ii) (1) Construct a truth table for the following statement formula : 5

$$(p \wedge \sim q) \vee (q \wedge (\sim p \vee r))$$

(2) Find the number of different non-equivalent statement formulas containing (A) one statement letter and (B) two statement letters. 5

(c) Answer either *(i)* or *(ii)* : 10

(i) (1) If two Boolean expressions are equivalent, show that their respective disjunctive normal forms contain the same terms. Find the complement of the following Boolean expression in disjunctive normal form : 3+2=5

$$xyz + x'yz + xy'z + x'y'z'$$

(2) Find a switching circuit which realizes the Boolean expression

$$x(y(z + w) + z(u + v))$$

Construct a truth table for the Boolean expression $x(y + x')$.

3+2=5

(ii) (1) Express the following Boolean expression in disjunctive normal form and conjunctive normal form in the variables present in the expression

$$(xy' + xz)' + x'$$ 5

(2) Find a switching circuit which realizes the Boolean expression

$$x + y(z + x'(t + z'))$$

Construct a switching table for the switching function represented by the Boolean expression $xy' + x'y$. 3+2=5

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