2014

MATHEMATICS

(Major)

Paper : 6.2

(Numerical Analysis)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) What is the normalized floating point representation of real numbers?

(b) Define the term relative error.

(c) If we take $\pi = 3 \cdot 14$ instead of 3·14159, find the absolute error.

(d) Establish the relation, $E \equiv 1 + \Delta$.

(e) Write down the value of $\Delta^{n+1} x^n$.

(f) When are Newton's interpolation formulae used?

(g) What is the basic principle of numerical differentiation?

2. Answer the following questions : 2×4=8

(a) Using normalized floating point representation, add ·4546E5 and ·5433E7.

(b) Show that

$$\Delta\{\log f(x)\} = \log\left[1+\frac{\Delta f(x)}{f(x)}\right]$$

(c) Establish the result

$$y' = \frac{dy}{dx} = \frac{1}{h}\left[\Delta y - \frac{1}{2}\Delta^2 y + \frac{1}{3}\Delta^3 y - \frac{1}{4}\Delta^4 y + \cdots\right]$$

where the symbols have their usual meanings.

(d) Write the conditions under which *(i)* trapezoidal rule and *(ii)* Simpson's $\frac{1}{3}$rd rule are valid.

3. Answer the following questions : 5×3=15

(a) Explain the terms truncation error and round-off error with suitable examples.

(b) Using the method of separation of symbols, prove that

$$u_1 x + u_2 x^2 + u_3 x^3 + \ldots = \frac{x}{1-x}u_1 + \frac{x^2}{(1-x)^2}\Delta u_1 + \frac{x^3}{(1-x)^3}\Delta^2 u_1 + \cdots$$

Or

Obtain the estimate of the missing figures in the following table :

x	:	1	2	3	4	5	6	7	8
$f(x)$	:	2	4	8	—	32	—	128	256

(c) Find the first derivative of the function tabulated below at the point $x = 3\cdot0$:

x	:	3·0	3·2	3·4	3·6	3·8	4·0
$f(x)$	:	−14·000	−10·032	−5·296	0·256	6·672	14·000

Or

Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using *(i)* Simpson's $\frac{1}{3}$rd rule and *(ii)* Simpson's $\frac{3}{8}$th rule.

4. Answer *either (a) or (b)* :

(a) *(i)* Derive the Newton's forward interpolation formula and mention when the formula gives best approximation. 4+1=5

(ii) In an examination, the number of candidates who obtained marks between certain limits were as follows :

Marks obtained	*Number of candidates*
0–19	41
20–39	62
40–59	65
60–79	50
80–99	17

Estimate the number of candidates who obtained less than 70 marks. 5

(b) (i) State and derive Stirling's central difference formula. Hence or otherwise establish Bessel's formula. 3+3=6

(ii) Apply Bessel's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$. 4

5. Answer *either (a) or (b)* :

(a) Explain briefly the idea of numerical integration. Establish the general quadrature formula and deduce trapezoidal rule from it. 2+5+3=10

(b) (i) The velocity v(km / min) of a vehicle which starts from rest, is given at fixed intervals of time t (min) as follows :

t :	2	4	6	8	10	12	14	16	18	20
v :	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes, using Simpson's $\frac{1}{3}$rd rule. 5

(ii) Compute the integral $\int_5^{12} \frac{1}{x} dx$ by applying Gauss's quadrature formula. 5

6. Answer *either (a) or (b)* :

(a) (i) State the Newton-Raphson formula and give a geometrical interpretation of it. 5

(ii) Find a root of the equation $x^3 - 4x - 9 = 0$, using the bisection method correct to four decimal places. 5

(b) (i) State the condition of convergence of Newton-Raphson method. 1

(ii) Derive $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ for determining the square root of $a > 0$, using Newton-Raphson formula. 2

(iii) Show that the equation $x^2 + \log x = 0$ has exactly one root and the root lies in the interval $[\frac{1}{3}, 1]$. 2

(iv) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places. 5

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