2014

MATHEMATICS

(Major)

Paper : 6.1

(Hydrostatics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) What do you mean by a surface of equal pressure?

(b) Define centre of pressure of a plane area immersed in a fluid.

(c) State the principle of Archimedes.

(d) What is centre of buoyancy?

(e) If the meta-centre is above the centre of gravity, what type of equilibrium can you expect?

(f) What is internal energy?

(g) Define the term convective equilibrium for a gas.

2. Answer the following questions : 2×4=8

(a) Show that the surface of equal pressure is intersected orthogonally by the lines of force.

(b) State the necessary and sufficient conditions of equilibrium of floating bodies.

(c) Define meta-centre and meta-centric height of a floating body.

(d) What do you mean by curves of floatation and curves of buoyancy?

3. Answer any *three* parts : 5×3=15

(a) If a mass of fluid is at rest under the action of given forces, obtain the equation which determines the pressure at any point of the fluid.

(b) Show that the position of the centre of pressure relative to the area remains unaltered by rotation about its line of intersection with the effective surface.

(c) Show that the equilibrium is stable or unstable according as the meta-centre is above or below the centre of gravity of the body.

(d) Prove that C_p is greater than C_v for a perfect gas, where C_p and C_v are the specific heat at constant pressure and volume respectively.

(e) For an adiabatic expansion, prove that $PV^{\gamma} =$ constant, where P is the pressure, V is the volume and γ is the ratio of specific heat at constant pressure and specific heat at constant volume for the gas concerned.

4. Answer *either (a) or (b)* : 10

(a) *(i)* Determine the necessary condition that must be satisfied by a given distribution of forces X, Y, Z so that the fluid may maintain equilibrium. 5

(ii) A liquid of given volume V is at rest under the forces $X = -\frac{\mu x}{a^2}$, $Y = -\frac{\mu y}{b^2}$, $Z = -\frac{\mu z}{c^2}$. Find the pressure at any point of the liquid and the surfaces of equal pressure. 5

(b) (i) If a fluid is at rest under the forces X, Y, Z per unit mass, find the differential equations of the curves of equal pressure and density. 5

(ii) Show that the forces represented by

$$X = \mu(y^2 + z^2 + yz)$$
$$Y = \mu(z^2 + x^2 + zx)$$
$$Z = \mu(x^2 + y^2 + xy)$$

will keep a mass of liquid at rest, if the density $\propto \frac{1}{(\text{distance})^2}$ from the plane $x + y + z = 0$; and the curves of equal pressure and density will be circles. 5

5. Answer *either (a) or (b)* : 10

(a) (i) A quadrant of a circle is just immersed vertically in a heavy homogeneous liquid with one edge in the surface. Determine the position of the centre of pressure. 5

(ii) If an area is bounded by two concentric semi-circles with their common bounding diameters in the free surface, prove that the depth of the centre of pressure is

$$\frac{3\pi(a+b)(a^2+b^2)}{16(a^2+ab+b^2)}$$ 5

(b) (i) A vessel in the form of an elliptic paraboloid, whose axis is vertical and equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{h}$$

is divided into four equal compartments by its principal planes. Into one of these, water is poured to the depth h; prove that, if the resultant pressure on the curved portion be reduced to two forces, one vertical and the other horizontal, the line of action of the later will pass through the point

$$\left(\frac{5}{16}a, \frac{5}{16}b, \frac{3}{7}h\right)$$ 5

(ii) A hollow hemispherical shell has a heavy particle fixed to its rim, and floats in water with the particle just above the surface and with the plane of the rim inclined at an angle 45° to the surface. Show that the weight of the hemisphere is to the weight of the water which it would contain $:: 4\sqrt{2}-5 : 6\sqrt{2}$. 5

6. Answer *either (a) or (b)* : 10

(a) *(i)* Show that the depth of the centre of pressure of the area included between the arc and the asymptote of the curve $(r-a)\cos\theta = b$ is

$$\frac{a}{4}\cdot\frac{3\pi a+16b}{3\pi b+4a}$$

The asymptote being in the surface and the plane of the curve is vertical. 5

(ii) A solid cone of semi-vertical angle α, specific gravity σ, floats in equilibrium in the liquid of specific gravity ρ with its axis vertical and vertex downwards. Show that the equilibrium is stable if $\frac{\sigma}{\rho} > \cos^6\alpha$. 5

(b) *(i)* When the temperature is supposed to be uniform, show that as the altitude increases in arithmetical progression, pressure decreases in geometrical progression. 5

(ii) The readings of a perfect mercurial barometer are α and β, while the corresponding of a faulty one, in which there is some air, are a and b. Prove that the correction to be applied to any reading c of the faulty barometer is

$$\frac{(\alpha-a)(\beta-b)(a-b)}{(a-c)(\alpha-a)-(b-c)(\beta-b)}$$ 5

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