2014

MATHEMATICS

(Major)

Paper : 5.6

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option for each of the following statements : 1×7=7

(a) The set of simultaneous equations

$$x_1 + 2x_2 = 8, 3x_1 + x_2 = 9, x_1 + x_2 = 4$$

is

(i) consistent

(ii) inconsistent

(iii) None of the above

(b) The set of points

$$(1, 3, 2), (1, -7, -8) \text{ and } (2, 1, -1)$$

in $\mathbb{R}^3$ is

(i) linearly dependent

(ii) linearly independent

(iii) None of the above

(c) A simplex in n-dimension is a convex polyhedron having

(i) $n-1$ vertices

(ii) n vertices

(iii) $n+1$ vertices

(iv) None of the above

(d) Let $X = \{x_1, x_2\} \subset \mathbb{R}^2$. Then the convex hull $C(X)$ of X is

(i) $\{\lambda x_1 + (1-\lambda)x_2 : \lambda \geq 1\}$

(ii) $\{\lambda x_1 + (1-\lambda)x_2 : \lambda \leq 0\}$

(iii) $\{\lambda x_1 + (1-\lambda)x_2 : 0 < \lambda < 1\}$

(iv) None of the above

(e) A linear programming problem (LPP) must have

(i) an objective (goal) that we aim to maximize or minimize

(ii) constraints (restrictions) that we need to specify

(iii) decision variables (activities) that we need to determine

(iv) All of the above

(f) At any iteration of the usual simplex method, if there is at least one basic variable in the basis at zero level and all $z_j - c_j \geq 0$, the current solution is

(i) infeasible

(ii) unbounded

(iii) non-degenerate

(iv) degenerate

(z_j, c_j having usual meaning)

(g) A feasible solution to an LPP

(i) must satisfy all of the problems constraints simultaneously

(ii) must be a corner point of the feasible region

(iii) need not satisfy all of the constraints

(iv) must optimize the value of the objective function

2. (a) Justify each of the following statements : 2×2=4

(i) The set

$$S = \{(x_1, x_2) : x_1, x_2 \in \mathbb{R}, x_1, x_2 > 0, x_1 x_2 \geq 1\}$$

is convex in $\mathbb{R}^2$.

(ii) Let S and T be two convex sets in $\mathbb{R}^n$. Then for any scalars $\alpha, \beta \in \mathbb{R}$, $\alpha S + \beta T$ is also convex in $\mathbb{R}^n$.

(b) Answer each of the following questions : 2×2=4

(i) Find the extreme points of the convex polygon defined by the inequalities

$$2x_1 + x_2 + 9 \geq 0$$
$$-x_1 + 3x_2 + 6 \geq 0$$
$$x_1 + 2x_2 - 3 \leq 0$$
$$x_1 + x_2 \leq 0$$

(ii) Show that the linear function

$$Z = CX,\ X \in \mathbb{R}^n,\ C \in \mathbb{R}$$

is a convex function.

3. Answer any *three* of the following : 5×3=15

(a) Find all basic feasible solutions of the system of equations

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 7$$
$$2x_1 + x_2 + x_3 + 2x_4 = 3$$

(b) Find graphically the maximum of $Z = 2x + 3y$ subject to

$$x + y \leq 30$$
$$y \geq 3$$
$$0 \leq y \leq 12$$
$$x - y \geq 0$$
$$\text{and} \quad 0 \leq x \leq 20$$

(c) Explain what you mean by feasible solution and basic feasible solution.

Describe briefly the method of reducing a feasible solution to a basic feasible solution of an LPP

$$\text{Max } Z = CX$$

subject to

$$AX = b$$
$$X \geq 0$$

under the assumption that the system has at least one feasible solution.

(d) If $x_1 = 1,\ x_2 = 2,\ x_3 = 1,\ x_4 = 3$ is a feasible solution of the set of equations

$$5x_1 - 4x_2 + 3x_3 + x_4 = 3$$
$$2x_1 + x_2 + 5x_3 - 3x_4 = 0$$
$$x_1 + 6x_2 - 4x_3 + 2x_4 = 15$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0$$

then find a basic feasible solution of the system.

(e) Prove that any convex combination of k different optimum solutions to an LPP

$$\text{Max } Z = CX, \quad C, X^T \in \mathbb{R}^n$$

subject to

$$AX = b$$
$$X \geq 0$$

is again an optimum solution to the LPP.

4. Prove that if either the primal or the dual problem of an LPP has a finite optimal solution, then the other problem also has a finite optimal solution.

Furthermore, the optimal values of the objective function in both the problems are the same, i.e.

$$\text{Max } Z_x = \text{Max } Z_w$$ 10

Or

Use simplex method to solve the LPP

$$\text{Max } Z = 4x_1 + 10x_2$$

subject to the constraints

$$2x_1 + x_2 \le 50$$
$$2x_1 + 5x_2 \le 100$$
$$2x_1 + 3x_2 \le 90$$
$$x_1, x_2 \ge 0$$

5. Use the two-phase simplex method to solve

$$\text{Max } Z = 5x_1 - 4x_2 + 3x_3$$

subject to the constraints

$$2x_1 + x_2 - 6x_3 = 20$$
$$6x_1 + 5x_2 + 10x_3 \le 76$$
$$8x_1 - 3x_2 + 6x_3 \le 50$$
$$x_1, x_2, x_3 \ge 0$$ 10

Or

Use duality to solve the LPP

$$\text{Max } Z = 2x_1 + x_2$$

subject to the constraints

$$x_1 + 2x_2 \le 10$$
$$x_1 + x_2 \le 6$$
$$x_1 - x_2 \le 2$$
$$x_1 - 2x_2 \le 1$$
$$x_1, x_2 \ge 0$$

6. A company has three plants at locations *A*, *B* and *C* which supply to warehouses located at *D*, *E*, *F*, *G* and *H*. Monthly plant capacities are 800, 500 and 900 units respectively. Monthly warehouse requirements are 400, 400, 500, 600 and 800 units respectively. Unit transportation costs (in rupees) are given below :

		To				
		D	*E*	*F*	*G*	*H*
From	*A*	5	8	6	6	3
	B	4	7	7	6	5
	C	8	4	6	6	4

Determine an optimum distribution for the company in order to minimize the total transportation cost. 10

Or

Solve the following assignment problem :

		Project			
		A	*B*	*C*	*D*
Engineer	*1*	12	10	10	8
	2	14	*Not suitable*	15	11
	3	6	10	16	4
	4	8	10	9	7

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