2014

MATHEMATICS

(Major)

Paper : 5.5

(Probability)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write the sample space for the experiment of tossing a coin three times in succession or tossing three coins at a time.

(b) What conclusion one can make about the conditional probability $P(A|B)$ if $P(B)=0$?

(c) Mention two properties which must be satisfied by the probability density function $f(x)$ of a continuous random variable X.

(d) State the multiplication theorem of expectation.

(e) State the relationship between the moment generating function of the sum of a number of independent random variables and the moment generating function of these individual random variables.

(f) If X is a random variable and a, b are constants, then express variance of $(aX + b)$ in terms of variance of X.

(g) Mention the relationship between the mean, median and mode of the normal distribution.

2. Answer the following questions : 2×4=8

(a) Show that probability of any impossible event is zero.

(b) If F is the distribution function of a random variable X and $a < b$, then show that

$$P(a < X \leq b) = F(b) - F(a)$$

(c) If X and Y are two random variables, then show that

$$\text{cov}(X, Y) = E(XY) - E(X)E(Y)$$

(d) If the random variable X follows binomial distribution with parameters n and p, then show that

$$E(X) = np$$

3. Answer any *three* parts from the following : 5×3=15

(a) If A and B are independent events, then show that $\bar{A}$ and $\bar{B}$ are also independent events.

(b) A bag contains 6 white and 9 black balls. Four balls are drawn at a time. Find the probability for the first draw to give 4 white and the second to give 4 black balls if the balls drawn in the first draw are not replaced before the second draw.

(c) A random variable X has the following probability function. Values of X—

x :	0	1	2	3	4	5	6	7
$p(x)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Find K and also evaluate

$$P(X < 6),\ P(X \geq 6) \text{ and } P(0 < X < 5)$$

(d) A coin is tossed until a head appears. What is the expectation of the number of tosses required?

4. Answer any *three* parts from the following : 10×3=30

(a) (i) A probability curve $y = f(x)$ has a range from 0 to ∞.

If $f(x) = e^{-x}$, find the mean and variance. 5

(ii) Let X be a continuous random variate with probability density function

$$f(x) = ax, \quad 0 \le x \le 1$$
$$= a, \quad 1 \le x \le 2$$
$$= -ax + 3a, \quad 2 \le x \le 3$$
$$= 0, \quad \text{elsewhere}$$

Compute $P(X \le 1 \cdot 5)$. 5

(b) (i) Show that expected value of X is equal to the expectation of the conditional expectation of X given Y i.e., $E(X) = E[E(X|Y)]$. 5

(ii) Discuss the effect of change of origin and scale on moment generating functions. 5

(c) (i) The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Find $P(X \ge 1)$. 5

(ii) Ten coins are thrown simultaneously. Use binomial distribution to find the probability of getting at least seven heads. 5

(d) (i) Show that the mean and variance of the Poisson distribution are equal. 5

(ii) Write the probability density function of a random variable X which follows Normal distribution with mean μ and variance σ^2. What is a standard normal variate? Find its mean and variance. 5

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