Total No. of printed pages = 7

3 (Sem 4) MAT M2

2015

MATHEMATICS

(Major)

Paper : M-4.2

Full Marks –80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following question : 1×10=10

(a) What is the physical significance of the moment of a force ?

(b) Under what conditions the effect of a couple is not altered if it is transferred to a parallel plane ?

(c) Define Angle of friction.

(d) What is the position of C. G. of a uniform parallelogram lamina ?

[Turn over

(e) When a body is said to be in unstable equillibrium ?

(f) What is Poinsot's Central axis ?

(g) State the principle of virtual work.

(h) Define amplitude and frequency of a Simple Harmonic Motion.

(i) What are the characteristics of a central orbit ?

(j) What do you mean by terminal vilocity ?

2. Answer the following questions : 2×5=10

(a) Prove that a single force and a couple in the same plane are equivalent to a single force, equal and parallel to the given single force.

(b) The algebraic sum of moments of a system of forces about four points whose coordinates are (x_1,y_1), (x_2,y_2), (x_3,y_3) and (x_4,y_4) referred to $\overrightarrow{OX}, \overrightarrow{OY}$ as rectangular axes are G_1, G_2, G_3, G_4 respectively. Show that

$$\begin{vmatrix} 1 & x_1 & y_1 & G_1 \\ 1 & x_2 & y_2 & G_2 \\ 1 & x_3 & y_3 & G_3 \\ 1 & x_4 & y_4 & G_4 \end{vmatrix} = 0$$

(c) State the laws of Friction.

(d) The speed v of a particle moving along x axis is given by the relation $v^2 = n^2 (8bx - x^2 - 12b^2)$ Prove that the motion is Simple Harmonic.

(e) Find the law of force towards the pole under which the curve $au = e^{n\theta}$ is described.
(The symbols have their usual meanings).

3. Answer the following questions : 4×5=20

(a) Prove that any system of coplanar forces acting on a rigid body can ultimately be reduced to a single force acting at any arbitrarity chosen point in the plane, together with a couple.

(b) A uniform ladder is in equillibrium with one end resting on the ground and other against a vertical wall. If the ground and wall be both rough, the coefficient of friction being μ and μ' respectively, and if the ladder be on the point of slipping at both ends, then show that the inclination of the ladder to the horizon is given by $\tan\theta = \frac{1-\mu\mu'}{2\mu}$.

(c) Find the C.G. of the arc of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ lying in the first quadrant.

Or

Find the C.G of the surface formed by the revolution of the cardoid $r=a(1+\cos\theta)$ about its axis.

(d) A body moving in a straight line OAB with Simple Harmonic motion has zero velocity at the points A and B whose distances from 0 are a and b respectively and has velocity V when half way between them. Show that the complete period is $\frac{\pi(b-a)}{V}$.

(c) If v_1 and v_2 are the linear velocities of a planet when it is respectively nearest and farthest from the sun, prove that $(1-e)v_1=(1+e)v_2$.

4. Answer the following questions : 5×4=20

(a) State and prove the necessary and sufficient conditions that a system of coplanar forces acting on a rigid body may be in equilibrium.

(b) Write down the forces which may be omitted in forming the equation of virtual work of a system of coplanar forces acting at different points of a rigid body.

(c) A point moving in a straight line with Simple Harmonic Motion has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is

$$2\pi\Big/\left(\frac{x_1^2-x_2^2}{v_1^2-v_2^2}\right)$$

(d) A particle is projected vertically upwards under gravity, supposed constant, in a resisting medium whose resistance varies as the square of the velocity. Show that the velocity of the particle in any position is given by

$$x=\frac{V^2}{2g}\log\frac{V^2+u^2}{V^2+v^2}$$

5. Answer the following :

(a) Obtain the differential equation of the path of a particle moving in a plane with an acceleration which is always directed to a fixed point in a plane. 5

(b) One end of an elastic string (modulus of elasticity λ) whose natural length is a, is fixed to a point on a smooth horizontal table and the other is tied to a particle of mass m, which is lying on the table. The particle is pulled to a distance from the point of attachment of the string equal to twice its natural length and then let go. Show that the time of complete oscillation is $2(\pi+2)\sqrt{(\frac{am}{\lambda})}$ 5

Or

A particle of mass m is travelling along x-axis such that at $t=0$, it is located at $x=0$ and has speed v_0. The particle is acted upon by a force which opposes the motion and has magnitude proportional to the instantaneous speed. Find the speed, position and acceleration of the particle at any time $t(>0)$.

6. Answer the following :

(a) Three forces act along the straight lines $x=0$, $y-z=a$; $y=0$, $z-x=a$; $z=0$, $x-y=a$. Show that they cannot reduce to a couple. 5

(b) A body rests in equillibrium upon another fixed body, the portions of the two bodies in contact have radii of curvature ρ_1 and ρ_2 respectively. The centre of gravity of the first body is at a height h above the point of contact and the common normal makes an angle α with the vertical. Prove that the equillibrium is stable or unstable according as

$$h < \text{or} > \frac{\rho_1 \rho_2}{\rho_1+\rho_2}\cos\alpha$$ 5

Or

A square of side : 2a is placed with its plane vertical between two smooth pegs which are in the same horizontal line at a distance c apart. Show that it will be in equillibrium when the inclination of one of its edges to the horizon is either 45^0 or

$$\frac{1}{2}\sin^{-1}\left(\frac{a^2-c^2}{c^2}\right).$$