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MATHEMATICS

(Major)

Paper : 2.2

(**Differential Equation**)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following as directed : $1 \times 10 = 10$

(a) The differential equation

$$\frac{dx}{dt} = \frac{x + 2xt + \cos t}{1 + t^2}$$

is

- (i) linear and not separable
- (ii) separable and not linear
- (iii) both separable and linear
- (iv) neither separable nor linear

(Choose the correct answer)

- (b) What is the integrating factor of the differential equation

$$\frac{dy}{dx} + \left(\frac{x}{1+x} \right) y = 1+x?$$

- (c) Write the standard form of homogeneous differential equation of order n .

- (d) What do you mean by self-orthogonal family of curves?

- (e) Write the particular integral of the differential equation

$$(D-2)^3 y = e^{2x}$$

- (f) Write down the general solution of the differential equation

$$y = px + ap(1-p)$$

- (g) Give the geometrical interpretation of the differential equation

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

- (h) Write the conditions for exactness of the differential equation

$$Pdx + Qdy + Rdz = 0$$

- (i) The partial differential equations can be formed by the elimination of

- (i) arbitrary constants only
 (ii) arbitrary functions only
 (iii) arbitrary functions or arbitrary constants
 (iv) None of the above

(Choose the correct answer)

- (j) Find an integral belonging to complementary function of the differential equation

$$(1-x^2)y_2 + xy_1 - y = x(1-x^2)^{3/2}$$

2. Answer the following questions : 2×5=10

- (a) Solve :

$$\frac{dy}{dx} = \sec(x+y)$$

- (b) Solve :

$$(D^3 - 4D^2 + 5D - 2)y = 0$$

- (c) Eliminate the arbitrary functions f and F from $z = f(x+iy) + F(x-iy)$.

- (d) Solve :

$$x dy - y dx - 2x^2 z dz = 0$$

(4)

- (e) Construct the partial differential equation by eliminating a and b from

$$z = ax + by + a^2 + b^2$$

3. Answer any four questions : 5×4=20

- (a) Prove that a necessary and sufficient condition that the differential equation

$$M dx + N dy = 0$$

be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

- (b) Find the orthogonal trajectories of the family of curves $3xy = x^3 - a^3$, a being parameter of the family.

- (c) What is a first-order differential equation? Reduce the following differential equation to linear form :

$$\frac{dy}{dx} + Py = Qy^n$$

Also write the integrating factor.

- (d) Solve by method of variation of parameter

$$y_2 + n^2 y = \sec nx$$

(5)

- (e) Solve :

$$z(xp - yq) = y^2 - x^2$$

- (f) Explain how to get general solution and singular solution of a differential equation of the form

$$y = px + f(p) \left(p = \frac{dy}{dx} \right)$$

4. Answer either (a) and (b) or (c) and (d) :

5+5=10

- (a) Show that the equation of the curve whose slope at any point is equal to $y+2x$ and which passes through the origin is $y = 2(e^x - x - 1)$.

- (b) Solve :

$$(x^3 D^3 - x^2 D^2 + 2xD - 2)y = x^3 + 3x$$

- (c) Solve :

$$(D^2 - 4D + 4)y = e^x + \cos 2x$$

- (d) Show that $Ax^2 + By^2 = 1$ is the solution of the differential equation

$$x \left\{ y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right\} = y \frac{dy}{dx}$$

Also write the order and degree of the differential equation.

(6)

5. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) Solve :

$$x^2 y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$$

(b) Remove the second term from the following equation and hence solve :

$$\frac{d^2 y}{dx^2} - \frac{2}{x} \frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right) y = x e^x$$

(c) Solve :

$$\cos x \frac{d^2 y}{dx^2} + \frac{dy}{dx} \sin x - 2y \cos^3 x = 2 \cos^5 x$$

(d) Solve the following equation, given that $\frac{\sin x}{x}$ is a part of complementary function :

$$\frac{d^2 y}{dx^2} + \left[1 + \frac{2 \cot x}{x} - \frac{2}{x^2}\right] y = x \cos x$$

6. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) Solve :

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$

(7)

(b) Find $f(y)$ such that the total differential equation

$$f(y) dx - xz dy - xy \log y dz = 0$$

is integrable. Find the corresponding integral.

(c) Solve :

$$\frac{d^2 x}{dt^2} - 3x - 4y = 0; \quad \frac{d^2 y}{dt^2} + x + y = 0$$

(d) Solve :

$$xz^3 dx - z dy + 2y dz = 0$$

7. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) Solve by Lagrange's method

$$(y^2 + z^2 - x^2)p - 2xyq + 2zx = 0$$

(b) Find the integral surface of the linear partial differential equation

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

which contains the straight line $x + y = 0, z = 1$.

(c) Solve by Charpit's method

$$pxy + pq + qy = yz$$

(d) Find the complete integral of

$$p^3 + q^3 = 27z$$

Find also the singular integral if it exists.
