3 (Sem 2) MAT M1

2015

MATHEMATICS

(Major)

Paper : 2.1

(Co-ordinate Geometry)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. (a) Transform the equation $y^2 - 4x + 4y + 8 = 0$ to parallel axes through the point (1, –2). 1

(b) What is the angle between the lines represented by the equation $x^2 - y^2 = 0$? 1

(c) What is the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1) ? 1

(d) What is the eccentricity of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (b > a) \ ?$$ 1

[Turn over

(e) Write down the asymptotes of the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 1

(f) What are the direction ratios of the normal to the plane 1

$x + y + z = 1$?

(g) Define skew lines. 1

(h) Write down the centre and radius of the sphere given by the equation 1

$x^2 + y^2 + z^2 - 2x - 4y + 6z = 22$

(i) What is the general equation of a second degree cone passing through the coordinate axes ? 1

(j) Define enveloping cylinder. 1

2. (a) Find the transformed equation of the line $y = x$ when the axes are rotated through an angle 45°. 2

(b) Find the value of K so that the following equation may represent pair of straight lines –

$Kxy - 8x + 9y - 12 = 0$ 2

(c) Find the equation of the cone whose vertex is at the origin and whose guiding curve is given by

$x = a, \quad y^2 + z^2 = b^2$ 2

(d) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are the extremities of any focal chord of the parabola $y^2 = 4ax$, prove that $t_1 t_2 = -1$ 2

(e) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the origin. 2

3. (a) Transform the equation

$3x^2 + 8xy + 3y^2 - 2x + 2y - 2 = a^2$ referred to a new set of axes through $(-1, 1)$ rotated through an angle $\pi/4$. 5

(b) Show that the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of parallel straight lines if

$\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$ 5

Or

Show that the straight lines joining the origin to the other two points of intersection of the curves whose equations are

$$ax^2 + 2hxy + by^2 + 2gx = 0 \quad \text{and}$$
$$a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$$
will be at right angles if $g(a'+b') = g'(a+b)$

4. (a) Prove that the line $lx + my = n$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if 5

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

(b) A sphere of constant radius r passes through the origin O and cut the axes at A, B and C. Prove that the locus of the foot of the perpendicular from O to the plane ABC is

$$(x^2 + y^2 + z^2)^2\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) = 4r^2$$ 5

Or

Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes

$y + z = 0, \quad z + x = 0, \quad x + y = 0, \quad x + y + z = a$

is $\frac{2a}{\sqrt{6}}$ and that the three lines of shortest distance intersect at the point $(-a, -a, -a)$

5. Answer any *four* parts : 5×4=20

(a) Show that the equation of the tangent to the conic

$$\frac{l}{r} = 1 + e\cos\theta$$

at the point whose vectorial angle is α is

$$\frac{l}{r} = e\cos\theta + \cos(\theta - \alpha)$$

(b) Reduce the equation
$$7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$$
to the standard form.

(c) Find the lengths of the semi-axes of the conic
$$ax^2 + 2hxy + ay^2 = d$$

(d) Prove that the equation of the polar of the origin with respect to the conic
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$
is $gx + fy + c = 0$

(e) Prove that the middle points of chords of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

parallel to the diameter $y = mx$ lie on the diameter $a^2my = b^2x$.

(f) Find the asymptotes of the hyperbola

$$2x^2 + 5xy + 2y^2 + 4x + 5y = 0$$

6. Answer any *four* parts : 5×4=20

(a) Find the condition that the two lines whose equations are

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$$

and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$

may intersect and also find the equation of the plane in which they lie.

(b) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

(c) Find the equation of the cylinder whose generators are parallel to the line

$$2x = y = 3z$$

and which passes through the circle

$$y = 0, \quad x^2 + z^2 = 8$$

(d) Find the equation of the tangent planes to $2x^2 - 6y^2 + 3z^2 = 5$ which pass through the line $x + 9y - 3z = 0, \quad 3x - 3y + 6z - 5 = 0$

(e) Find the condition that the plane

$$lx + my + nz = p$$

may be a tangent plane to the conicoid

$$ax^2 + by^2 + cz^2 = 1$$

(f) Find the condition that the line

$$\frac{x - 2}{l} = \frac{y - 1}{m} = \frac{z - 3}{n}$$

may touch the ellipsoid

$$3x^2 + 8y^2 + z^2 = c^2.$$