2014

MATHEMATICS

(Major)

Paper : 2.1

(Coordinate Geometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1\. *(a)* What will be the equation of the line $ax+by+c=0$, if the origin is transferred to the point (α, β)? 1

(b) What is the locus represented by the equation $x^2-5xy+6y^2=0$? 1

(c) About which axis the parabola $y^2=4ax$ is symmetric? Justify your answer. 1

(d) The parametric equations $x=a\sec\phi$, $y=b\tan\phi$ represent *(i)* an ellipse, *(ii)* a parabola, *(iii)* a hyperbola.

Find the correct answer from above. 1

(e) Write the relationship between the lengths of semi-major axis, semi-minor axis and the eccentricity for the standard equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \; b > a$$ 1

(f) What are the direction cosines of the normal to the plane given by the equation $2x - 4y + 3z = 9$? 1

(g) Find the radius and centre of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$$ 1

(h) What is the general equation of a second-degree cone passing through the coordinate axes? 1

(i) The shortest distance between the two lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

is given to be zero. What conclusion can you make about the lines? 1

(j) What are the basic natures of the guiding curve and the generator for a right-circular cylinder? 1

2. *(a)* Prove that the equation

$$2x^2 - 5xy + 3y^2 - 2x + 3y = 0$$

represents two lines. 2

(b) Prove that the equation

$$y^2 + 2ax + 2by + c = 0$$

represents a parabola, whose axis is parallel to the axis of x. 2

(c) Find the equation of the plane through the point (2, 3, 5) and parallel to the plane $2x - 4y + 3z = 9$. 2

(d) Find the equation of the right-circular cone whose vertex is the origin, axis is the z-axis and semi-vertical angle is α. 2

(e) The axis of a right-circular cylinder is

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{5}$$

and its radius is 5. Find its equation. 2

3. *(a)* If by a rotation of the rectangular axes about the origin, the expression $ax^2 + 2hxy + by^2$ changes to

$$a'x'^2 + 2h'x'y' + b'y'^2$$

then show that

$$a + b = a' + b'$$

$$ab - h' = a'b' - h'^2$$ 5

(b) Find the condition that the general equation of second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of straight lines. 5

Or

Prove that the straight lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

will be equidistant from the origin if

$$f^4 - g^4 = c(bf^2 - ag^2)$$

4. *(a)* Find the equation of the plane which passes through the point (2, 1, 4) and is perpendicular to the planes

$$9x - 7y + 6z + 48 = 0$$
$$x + y - z = 0$$
5

(b) Prove that the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar. 5

Or

Find the length and equations of the line of the shortest distance between the lines

$$\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$$

$$\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$$

5. Answer any *four* parts : 5×4=20

(a) Find the equation of the pair of tangents from (x', y') to the parabola $y^2 = 4ax$.

(b) If the line $\frac{lx}{a} + \frac{my}{b} = n$ cuts the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the ends of a pair of conjugate diameters, prove that $l^2 + m^2 = 2n^2$.

(c) Prove that the middle points of chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ parallel to the diameter $y = mx$ lie on the diameter $a^2my = b^2x$.

(d) Prove that from any point six normals can be drawn to the conicoid

$$ax^2 + by^2 + cz^2 = 1$$

(e) Prove that the equation of the polar of the origin with respect to the conic $ax^2+2hxy+by^2+2gx+2fy+c=0$ is $gx+fy+c=0$.

(f) Find the condition that the line $lx+my+n=0$ is a tangent to the conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

6. Answer any *four* parts : 5×4=20

(a) Find the equations of the tangent planes to the sphere $x^2+y^2+z^2=49$ which passes through the line

$$2x+z-21=0=3y-z+14$$

(b) Show that the equation of the cylinder whose generators are parallel to the line

$$\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$$

and guiding curve is $x^2+2y^2=1,\ z=3$ is

$$3(x^2+2y^2+z^2)+8yz-2zx+6x-24y-18z+24=0$$

(c) Prove that the section of a cone with vertex P and guiding curve the ellipse $z=0,\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by the y-plane is a rectangular hyperbola.

(d) Find where the line

$$\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z+3}{4}$$

meets the plane

$$2x+4y-z+1=0$$

(e) Find the equations of the tangent planes to the conicoid $2x^2+3y^2-4z^2=1$ which are parallel to the plane $x-3y+z=0$.

(f) Show that any normal to the conicoid

$$\frac{x^2}{pa+q}+\frac{y^2}{pb+q}+\frac{z^2}{pc+q}=1$$

is perpendicular to its polar line with respect to the conicoid

$$\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}=1$$

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