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MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Write down the nth derivative of $\cos(3x+5)$.

(b) If $u = f\left(\frac{y}{x}\right)$, then obtain the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(c) The equation of a curve is $\log y = x \log a$. What is the length of the subtangent to the curve at the point $P(x, y)$?

(d) Define the curvature of a curve at point on it.

(e) Write down the equation of the asymptote of the curve $xy - 3x - 4y = 0$ which is parallel to the x-axis.

(f) If $f(x, y) = \log(x^2 + y^2)$, then determine $\frac{\partial f}{\partial y}$.

(g) Choose the correct answer :

$\int \sqrt{a^2 - x^2}\, dx$ (ignoring the constant of integration) equals

(i) $\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\cos^{-1}\frac{x}{a}$

(ii) $\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

(iii) $\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right)$

(iv) $\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)$

(h) Write down the value of $\int_{-a}^{a} x^3 f(x)\, dx\, (a \neq 0)$ where f is an even function.

(i) Evaluate $\int_{-\pi/2}^{\pi/2} \cos x\, dx$.

(j) Write down the intrinsic equation of the catenary $y = c\cosh\left(\frac{x}{c}\right)$.

2. Answer the following questions : 2×5=10

(a) If $y = \log(ax + x^2)$, then find y_n.

(b) Find $\frac{ds}{d\theta}$ for the curve $r = ae^{\theta \cot \alpha}$.

(c) Show that

$$\int_0^{\pi} xf(\sin x)\, dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$$

(d) Show that the area of a loop of the curve $r = a\cos 2\theta$ is $\frac{\pi a^2}{8}$.

(e) Find the volume generated by revolving about OX, the area bounded by $y = x^3$ between $x = 0$ and $x = 2$.

3. Answer the following questions : 5×4=20

(a) If $u = F(y - z, z - x, x - y)$, then prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

Or

If $y = f(x + ct) + \phi(x - ct)$, then show that

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

(b) Trace the curve $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$; $-\pi \le \theta \le \pi$.

Or

Prove that the sum of the intercepts of the tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ upon the coordinate axes is constant.

(c) Evaluate :

$$\int \frac{dx}{3+5\cos x}$$

Or

$$\int \frac{dx}{(1+x)\sqrt{1+2x-x^2}}$$

(d) Find the perimeter of the circle $x^2+y^2=a^2$.

4. Answer *either* (a) *or* (b) :

(a) State the Leibnitz theorem and use it to prove the following : 2+5+3=10

(i) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$ where $y=(\sin^{-1}x)^2$

(ii) $y_n = \dfrac{(n-1)!}{x}$ if $y = x^{n-1}\log x$

(b) Define homogenous function and its degree. Also answer the following : 2+4+4=10

(i) If u is a homogeneous function in x and y of degree n having continuous partial derivatives, then prove that

$$\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)^2 u = n(n-1)u$$

(ii) If $u=\cos^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, then show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\cot u=0$$

5. Answer *either* (a) *or* (b) :

(a) (i) Find the asymptotes of the curve $x^4-x^2y^2+x^2+y^2-a^2=0$.

(ii) If ρ_1 and ρ_2 are the radii of curvature at the ends P and D of conjugate diameters of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

show that

$$\rho_1^{2/3}+\rho_2^{2/3}=\frac{a^2+b^2}{(ab)^{2/3}}$$ 5+5=10

(b) Define cusp and isolated points. Search for double points on the curve

$$x^2y + x^3y + 5x^4 = y^2$$ 2+8=10

6. (a) If $u_n = \int_0^{\pi/2} \theta \sin^n \theta \, d\theta$ and $n > 1$, then prove that

$$u_n = \frac{n-1}{n} u_{n-2} + \frac{1}{n^2}$$ 5

(b) If $I_n = \int (a^2 + x^2)^{n/2} \, dx$, then show that

$$I_n = \frac{x(a^2 + x^2)^{n/2}}{n+1} + \frac{na^2}{n+1} I_{n-2}$$ 5

7. (a) Find the area of the region bounded by the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$. 5

(b) Find the surface area of the solid generated by revolving the cardioid $r = a(1 - \cos\theta)$ about the initial line. 5

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